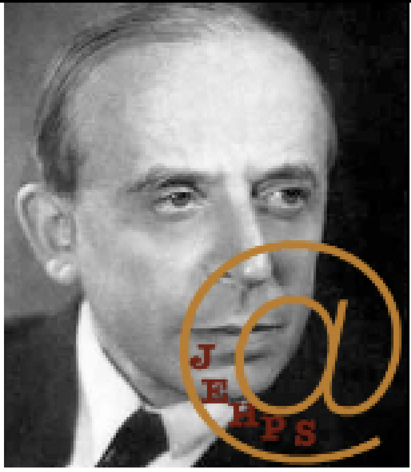

Journ@l Electronique d'Histoire des Probabilités et de la Statistique

Electronic Journ@l for History of Probability and Statistics

Vol 3, n°2; Décembre/December 2007

www.jehps.net

An introduction to Karel Vorovka's philosophy of randomness

(followed by two translations from the Czech)

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Résumé

Cet article contient une brève présentation des travaux du philosophe tchèque Karel Vorovka (1879-1929) en philosophie des sciences, suivie de la traduction d'une conférence sur Vorovka par Bohuslav Hostinský et d'un article mathématique de Vorovka.

Abstract

We give a short presentation of the Czech philosopher Karel Vorovka (1879-1929) 's work in philosophy of science, followed by the translation of a conference on Vorovka by Bohuslav Hostinský and of aVorovka's mathematical paper.

Keywords and phrases : Probability, randomness, philosophy of science

AMS classification :

Primary : 00A30

Secondary : 01A60, 60-03

Introduction

The aim of this introduction is to provide some information on an important figure on the Czech philosophical scene at the beginning of the 20th Century. Karel Vorovka, born in 1879, played a remarkable rôle in the philosophical interpretation of sciences. In particular he was one of the first philosophers to try to interpret Poincaré's writings and to use them to develop a modern conception of the interaction between man's free will and the constraints implied by the laws of science.

Vorovka remains largely unknown outside the Czech lands, for two reasons. Firstly he died prematurely, at the age of 50, and therefore he had not time enough to collect his ideas into a large-scale work. His writings are not extensive: a few books and a few papers. Secondly and

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more importantly, most of his writings are in Czech and have never been translated. A few were originally published in German, French or Italian when Vorovka participated in international congresses. At this point we may mention our plan to fill this gap-at least partly-by arranging the translation of some of his major works over the next few years.

What is very noticeable in Vorovka's approach is how it was clearly inspired by his mathematical background as well as his religious approach to the world. In this respect, he may be seen as an heir to Bernhard Bolzano (1771-1848), the main figure on the Prague philosophical and mathematical scene of the 19th Century. Vorovka's discovery of Poincaré's epistemological texts at the turn of the Century had a decisive influence on his thinking. After reading Poincaré, it became clear to him that, at a moment when sudden changes (especially in Physics) were challenging the foundations of our understanding of the external world, there was a need for mankind to redefine its place in the world and to re-examine the nature of man's will.

While he was reading Poincaré, Vorovka (as he appears in Hostinský's text presented below) appears to have been impressed by the role given to randomness and probability calculations in the new scientific approach to the world. Vorovka did not limit himself to philosophical considerations but was anxious to understand the technical aspects and recent developments. We present below one of Vorovka's rare mathematical works, published in 1912. It is devoted to an extension of a 1905 paper by Markov ([6]) treating the ruin of two gamblers playing together; this was itself inspired by Bertrand's considerations in his famous textbook [1] (on the latter, the reader may consult [3] with profit). In his paper, Markov tries to compute the probabilities of ruin of both gamblers, that is, when they might need to stop the game because they do not have the necessary minimum stake to continue wagering. The Russian mathematician makes use of difference equations obviously connected with what would soon be described as a chain property (Markov's original paper was written in 1906 - see [7]) and their hitting times. As the explicit solutions to the equations may be hard to express, Markov proposes approximate values for the ruin probabilities. The objective of Vorovka's short note is to prove that with probability one, one of the two gamblers will eventually be ruined.

The fact that he was in position to quote Markov's text, published in Russian in Kazan (like the first paper on chains), indicates that Vorovka followed the latest developments of probability theory. We have not been able to discover how this came about, whether he read the texts on his own or learnt about them from his contacts mathematician colleagues at Prague university? However Vorovka learnt of this technical work, he seems to have worked on these problems only to familiarize himself with the mathematics involved. His main concern was to understand the underlying meaning of probability calculations and how the consideration of a quantified randomness could play a role in the link we have mentioned between man's will and a purely logic and scientific description of the world.

In Poincaré, Vorovka found an overriding concern with preserving a unified understanding of the world, in particular in Physics (see the chapters of Part IV of [10]). As Poincaré writes (Chap.IX, p.161): *We do not have to ask* **whether** *nature is one, but* **how** *it is one*[2]. Vorovka tries to keep this idealistic tendency when man's liberty of action is included. In a text published two years before he died in the proceeding of the Boston 6th international Congress of philosophy of 1926, Vorovka partly delivers a solution he obtained for the problem. In order to make precise the role of man's action in the world, it is desirable to introduce a quantification of these actions through the concept of *sufficient action*. Vorovka considers the following example : when an electric bell

[2] our emphasis.

is released by a button, the mechanical pressure on the button can be unsufficient to activate the bell. Vorovka postulates that in every experimental science there intervenes a quantifying concept linking the change in the system state and the action. Vorovka therefore proposes to change Kant's basic definition of causality in order to take into account the complexity of the systems acting in the world. More precisely, Vorovka proposes to replace Kant's classical assertion *Any change presupposes something from where it ensues in accordance with a rule* by the less rigid assertion *If the regular course of a material system is subject to a change, this change is due to the anterior action of another system from where it ensues in accordance with a rule*. As Vorovka comments, this modification emphasizes that the world is composed of innumerable systems, sometimes very complex ones such as living organisms. We may not be aware of the individual characteristics of the elements involved in the systems. However, the similarities we observe between complex systems (such as the organisms) frequently make prediction possible. One may certainly draw a parallel between this kind of approach and Poincaré's line of thought about the origins of Mathematical physics in [10], for example when he comments on the relations between macro and microscopic scales (p.170) : *The simplicity of the elementary phenomenon was hidden under the complexity of the resulting observable phenomenon; but, in its turn, this simplicity was only apparent and concealed a very complex mechanism* or on the fact that in Physics, generalizations are often presented through mathematical equations (p.171) : *it is not only because one has to express numerical laws ; it is because the observable phenomenon is due to the superposition of a large number of elementary phenomena* similar one to another*; therefore differential equations naturally appear.* [3].

Several tributes published after his death testify to Vorovka's influence on Czech mathematicians. One of those he influenced was Bohuslav Hostinský, who in the 1930s became a leading specialist in the theory of Markov chains (see [2], [4], [5]). We give below a translation of an important lecture given by Hostinský at the Czechoslovak union of mathematicians and physicists in 1929. Hostinský recalls in particular how Vorovka explained Poincaré's conception of randomness to him before World War I. We have described in [4] how Hostinsk ý began his studies in probability soon after. Let us quote also the testimony of Bohumil Bydžovský writing in the small book edited by the Prague philosopher Ferdinand Pelikan in honor of Vorovka ([9], p.61). In another publication ([8], pp. 15-16), Pelikan had already presented Vorovka's important role in Czechoslovakian philosophy as follows.

> Vorovka's work constitutes a powerful contribution to modern idealism in Czechoslovakia. It provoked an energetic reaction against the positivistic trend. The two trends devide when F.Pelikan created his journal *The Philosophical Motion*[4](1919). Opponents gathered around another and older journal, *the Czech thought*[5] (created at the begining of the XXth Century). Under the aegis of Prof.Krejči, this journal became the organ of the positivists and the realists.

Pelikan especially emphasized the last book Vorovka published in his life-time as a good digest of his thought. Let us finish this short presentation with Pelikan's comments on this work.

> The book by Karel Vorovka, *Skepticism and Gnosticism*, published in 1922, is a very representative work of the new idealistic trend in Czechoslovak thought. In

[3]Poincaré's emphasis
[4]Ruch Filosofitský
[5]Česká mysl

it one may find a movement towards an emancipation from philosophical skepticism as well as from contemporary dogmatic positivism, close to scholasticism. Vorovka is before anything a mathematician and there is nothing surprising in the fact that he shows himself especially receptive to the beauty and the harmony of the quantitative relations (in other words, to numbers), relations in which he thinks to observe an ideal union of the true and the beautiful. He is an idealist (as he himself admitted) ; he is surprised, charmed by the harmonious macroscopic connexion characterizing our solar system. Vorovka believes in the objective existence of an ideal, located in Plato's 'realm of ideas', but whose real existence on earth is also obvious in the subconscious psychological strengths of men. Victory over skepticism, faith in ideal, which is to say in a science continuously renewing its methods, confidence in the mind and its advances - here are the fundamental ideas on which Vorovka has built his work. Anyone who is in position to distinguish, to catch the central meaning of his own life, to penetrate in the depths of individualism, will agree with Vorovka in asserting that it is precisely there that one must seek the crucial point where lies any human knowledge. Its fire, in the form of the manifestation of genius, sometimes lights the world up.

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Annex 1

Vorovka's activities in the philosophy of mathematics

Bohuslav HOSTINSKÝ

Translation from the Czech by Stepanka Bilova and Laurent Mazliak

The origin for Vorovka's writings come from his studies at the Prague Faculty of Arts. He studied mathematics and physics there and graduated in 1901. Afterwards he soon published his first work: Particular integral as an envelop. (Časopis pro pěstovánímatematiky a fysiky, XXXII, 1903.) It is a critical study devoted to a special problem of differential calculus. Vorovka thoroughly analyzed one paragraph of the German translation of famous Serret's book on differential and integral calculus which is not present in the French original paper. He shows that this part contains mistakes and corrects them. He also wrote a fine paper about related considerations on envelope curves which was published in the annual report of Vinohradská secondary school 1902-1903.

However, mathematical questions in the narrowest sense of this word did not remain Vorovka's speciality, his interests turned to the philosophical side of mathematics. We have to mention Poincaré's influence as a source that inspired him when choosing mathematical-philosophical topics and showed him ways of solving problems. Vorovka's article titled **Konvencionalism** *(Conventionalism)* (Česká Mysl, X, 1909) begins with these words: "The following considerations arouse while I was reading the book **La valeur de la Science**. In this book, as well as in his earlier work **La Science et l'Hypothèse**, the renowned mathematician Henri Poincaré brought a number of enriching philosophical ideas and he presented them in such a magnificent form that no mathematician or philosopher would set those books aside without a feeling of gratitude to the famous author. These works are written with a particular sincerity. The author does not impose his opinions, but proposes all possible objections against them. When the reader supposes to have absolute certainty and to master the final solution of a problem, Poincaré presents new unexpected doubts at the end of the chapters, and this always gives an impulse to think. The present text was written exactly in this way." Next, Vorovka presents the principle of Poincaré's conventionalism: fundamental mathematical theorems and definitions are to a certain extent arbitrary, but not completely, because our experience guides our choice of conventions. The following sentences (mentioned paper, p. 226) seem to be particularly characteristic of how Vorovka understood scientific problems: "Things are immensely complicated and natural events are also much more complicated than everything that we suppose to learn on them. And also our ideas, as being natural events, are much more complicated than what we are able to detect in them and to express." This is the opinion which Vorovka repeats in his later works and which we heard personally from him in conversations and discussions. In 1912, there was a change in regulations of the Union of Czech Mathematicians and Physicists. Among others it was decided to establish a special scientific board whose main aim was to improve lectures organizing. The board was to have two sections: mathematical and physical. As a member of the committee I proposed Vorovka as a candidate for mathematical-philosophical questions in the mathematical section. Vorovka, who was then a professor at the Prague Old-Town Secondary School, became a member of the scientific board and his work in the Union was outstanding. Mathematical lectures were given in a lecture-room of the Mathematical institute in Karlov. After he read lectures himself or took part in discussions, he gained the audience favor for his interesting and deeply worked out presentation of difficult questions. In July 1912, H. Poincaré died and the first activity of the scientific board, elected in December 1912, was a series of lectures on various fields of the great mathematician's activities. Vorovka was asked to speak on the topic H. Poincaré as a Philosopher; the lecture was read on January 25th , 1913 and a part of it was adapted and published under the following title: **Jak soudil H. Poincaré o vztazích matematiky k logice** *(H. Poincaré's Opinions on the Relationships of Mathematics and Logic)*. (Časopis pro p. m. a f., XLIII, 1914.) The title indicates the questions which interested Vorovka in this paper. I shall return to them later. But now, I mention his considerations about probability calculus. Vorovka had deep knowledge even in this field, as it is shown in his studies called **Poznámka k problému**

ruinováníhráčů *(A note to the problem of gamblers ruin)*(Časopis pro p. m. a f., XLI, 1912) and **O pravděpodobnosti příčin** *(On the probability of causes)* (Časopis pro p. m. a f., XLIII, 1914). But what is especially valuable is his work on philosophical significance of probability calculus whose content and methodology are also inspired by Poincaré.

The most significant of all Poincaré's mathematical-philosophical considerations is perhaps his analysis of the concept of randomness (**La science et l'hypothèse**, chap. XI; Science et Methode, chap. IV; Calcul des probabilities, 2e edition) and related probability evaluation. I am happy and grateful to remember the conversations I used to have on these matters with Vorovka in 1912 and the following years. In 1913, his paper **Filosofický dosah počtu pravděpodobnosti** *(Philosophical significance of probability calculus)* (Česká Mysl, XIV, 1913) was published. There, Vorovka analyses various opinions on the meaning of probability calculations. Especially is it possible that through calculus we increase the hope that some event will occur again because we know that it has occured many times. Every use of probability calculus is based on some regularity which we observe in the results of statistics. Thus, e. g. if we throw a die 6000 times, "one" will be obtained approximately 1000 times, "two" as well and so on. The statistics of throws shows that each of the six numbers will occur in the series of 6000 experiments approximately the same number of times. This is a known result. Some philosophers, who are inclined to think that science is nothing else than a collection of facts, are satisfied with this statistical statement.

But others share the opinion that it is possible to go further and to grasp the understanding why the results in that series of 6000 throws are divided approximately equally among six numbers. On which assumptions such a deduction may be based? The older standpoint, which corresponds to the classical probability calculus, can be expressed in this way: A die is perfectly symmetric; no side can be ascribed any special properties which cannot be ascribed to every other side at the same time. It is understood that we suppose not only a perfect geometrical shape of the die, but also an even distribution of the mass inside it. Thus as no side of the die can be distinguished from another side, one must consider the six possible issues as equally possible. This principle, which leads to good results in many cases, is logically incorrect because while looking for probabilities of various results we suppose they are the same; it is an obvious circulus vitiosus. Therefore this principle of "equally probable results" has been unfavorably criticized for a long time and also Poincaré disapproved of it. Thus we refuse it because there is no other possibility. But then the situation seems to be hopeless. What should the deduction we are looking for be based on? Poincaré, however, showed that the problem can be solved. The solution is based on the analysis of the objective conditions of randomness. According to Poincaré, random events have two characteristic properties. First, they are instable : a small change in the cause is related to a big change in the effect (in our case: if I take a die into my hand in a certain way and then throw it in a certain direction with a certain speed, the die moves on the table for a while and then eventually stops on the table in a certain position. However, if I change either the initial position of the die or the speed given at the initial moment only a little bit, the result of the throw will be quite different from before. The other basic property of random events is the complexity of conditions determining it (in our case: the movement of the die on the table is complicated). What is certain is that the initial position of the die and its initial speed determine its final position. But how should this be taken into account? Poincaré managed to perform a calculation in some simple cases from which I present only one. A player shuffles cards and repeats shuffling many times. What is the expected probability of various arrangements of cards at the end? The way of shuffling is given by the player's personal habit; one player shuffles cards in one way, another one in a different way. The calculation, however, shows that after shuffling the pack many times, all arrangements of cards have the same probability independent of the player's habits. The player's individuality is mirrored in some regularity which he follows more or less consciously when he shuffles cards; thus we find that the player's individuality does not influence the final result. In accordance with this ingenious example we can set up a deductive determination of probability independent of the unacceptable hypothesis of "cases equally probable". Vorovka correctly described the impact of Poincaré's idea that an assumption for such calculations takes always the form of some regularity. He states at the end of his beautiful study: "This regularity and determinism of all phenomena are in fact a postulate for any scientific research, it is the basic assumption of physics, chemistry as well as probability calculus." In other words: probability calculus cannot be used to prove laws of natural events.

12 years ago I began to deal with Poincaré's methods in probability calculus and at the beginning I did not find it clear whether, in more complicated cases, we can use a procedure similar to the one successful in simple ones. Now I know that his methods are really general, and all what I think about the foundation of these questions is in complete agreement with the ending of Vorovka's paper written 16 years ago. Vorovka is certainly one of the first (and not only among Czech) writers who fully appreciated the significance of Poincaré's considerations on the concept of randomness.

It is not surprising that Vorovka, having seen how Poincaré presented his profound and fruitful ideas simply, with a transparent logic and without developing new terminology, could not give a favorable opinion on a paper like Meinong's **Möglichkeit und Wahrscheinlichkeit** which he criticized for its poor intellectual content. **Počet pravděpodobnosti v novější filosofii** *(Probability Calculus in New Philosophy)*, a lecture given on January 15th, 1920 in the Union of Czechoslovak mathematicians and physicists - the content is stated briefly in the annual report of the Union 1919-20, p. 9)

Now, I come to the topic which Vorovka was greatly engaged in, i.e. specifying the role of logical elements and the role of intuitive elements in mathematics. We meet logic and views in mathematics, in reasonings as well as in proofs, and also in applications of mathematics to physical problems and others. Apart from the lecture on Poincaré, which I have already mentioned, Vorovka gave three other lectures on this topic at the Union. On November 27th, 1915 he lectured **O logických teoriích matematiky***(On logical theories of mathematics)*, on December 11th, 1915 **O logických kalkulech***(On logical calculi)* and on November 12th, 1916 **O vztazích mezi Leibnizovou filosofiía matematikou***(On relations between Leibniz's philosophy and mathematics)*.
What is intuition in mathematics? By this word, we do not mean an opinion obtained through the senses, but some abstract knowledge. To make it clear, I will state what Vorovka writes according to the above mentioned Poincaré's paper (Časopis pro p. m. a f., XLIII, p. 156). "Poincaré clarifies this more or less uncertain idea of intuition by providing psychologically interesting records from works of famous mathematicians, and he also likes to return several times to the comparison with a game of chess. In order to understand this game, it is not sufficient to check the correctness of the moves, according to the given rules; to understand a player's strategy, to comprehend the purpose of his moves, we must see the aim to which the game is leading from a distance. In quite a similar way when we prove something in mathematics, it is not enough to find the conclusions logically correct, but it is necessary to understand why the definitions, conclusions, theorems are ordered in that particular way, not in another one. This organic unity can be perceived in one single bright moment, and if this ability is necessary for studying a complete proof, it is even more

required for finding new truths. Both a mere empiricism and a mere logic would be only groping in the dark if they were not associated with the most intellectual, the most internal power of genius, often opposing all senses, a power which moves the whole mechanism of logic and which is perhaps the very true intellect : that is intuition." In that way we have an intuition of a genuine number, or an intuition that the same event can be repeated infinite many times if it has been only once possible, and so on. The disputes that have been held on the relationship between views (intuition) and logic have a deep root according to Poincaré. It is a dispute between logicism and psychologism. Vorovka presented his opinion on these problems in a large study published under the title **Úvahy o názoru v matematice** *(Considerations on an Opinion in Mathematics)* by the Ist and IInd Class of the Czech Academy in 1917. Logicism is a field which recognizes the concept of "autonomous truth" and acknowledges that such truths (e. g. abstract mathematical propositions) exist out of time and space and out of human consciousness. If we manage to prove a mathematical theorem, at least if we incline to logicism, we think that it is valid not only for this moment but for all times, present as well as past. In fact, it is not possible to create a truth, it is only possible to discover it, like Columbus discovered America. Plato's theory of ideas is related to this opinion. Leibniz is one of his most important supporters; with enthusiasm, he glorifies the importance of syllogism for discovering new truths and he is convinced of the omnipotence of logic. In a more modern time, the English philosopher of mathematics Russell worked out these ideas to extremes. Russell explicitely says that "Logic and mathematics force us, then, to admit a kind of realism in the scholastic sense, that is to say, to admit that there is a world of universals and of truths which do not bear directly on such and such a particular existence...Thus, the understanding spirit of these truths is something quite subsidiary for mathematics and for the righteousness of its propositions." However, in such a philosophy, Vorovka says, there is no place for an active creative mind.

Unlike logicism, psychologism does not acknowledge autonomous truths. According to Vorovka, questions on the relationship of logicism and psychologism must be considered by anybody interested in the connection between abstract science and concrete science. Mathematics can be applied to the world of senses. Every logicism must necessarily follow Leibniz's footsteps and be supported by some construction, in the form of prestabilized harmony, as soon as it tries to explain the possibility of this mathematical application. To clarify both viewpoints in more detail, Vorovka analyses and criticizes various logic theories. He objects to traditional formal logic that it does not sufficiently take into account the fact that most mathematical judgement are judgements on relationships. Such a judgement does not subordinate one concept to another, but stresses only a byproduct which is implied from the comparison of both concepts. He presents various systems of logic in the next chapter where he uses symbols in a way similar to algebra and the turns to the psychology of thinking. Vorovka writes: "There must be a connection as narrow as possible between thinking and reality so that there would not be necessary to build any superficial bridge of prestabilized harmony at the end." There should not arise any conflict between abstractions and perceptions. In the general acception, a variable symbol is not totally omitted or denied. If I imagine metal in general, I do not imagine it without a color, weight, hardness, but with some non-specified color, with some non-specified weight etc. General ideas in various minds can have various values depending on how complete and ordered series of single ideas are. Our general idea of a triangle is in fact only a state which is our ability to go through a series of single ideas which are connected to the general idea of triangle. Then, if judgements deal with general ideas only, the conviction of their correctness can never be imminent, but must be proved. A rigorous psychologism cannot allow the so-called principle of identity to have such an importance that is given to it by supporters of schools refering to logicism. The meanings of the words ”the same” and ”different” are related; one cannot be understood without the other. And it is possible to understand them only when a person has gained, by his thinking, sufficient inner experience about real objects. Now I am closing this brief survey of Vorovka’s characteristics of logicism and psychologism. It however gives a brief account of his interesting and subtle considerations on mathematical thinking presented in his work. Vorovka professes psychologism. ”We admitted,” he writes at the end of his work, ”that forms cannot be perfectly separated from substance É The junction in human thinking, in other words the conviction that the truth is communicable and imposes itself to others, is based not only on the form being identitcal, but also on the substance being identical.” At the same time Vorovka is, however, absolutely fair towards logicism. He acknowledges the significance of formal elements and later tries to specify exactly what he had found right in Poincaré’s criticism of logicism (**Ce qu’il y a de juste dans la critique de la logistique de H. Poincaré**; Atti del V Congresso Internazionale di Filosofia, Napoli, 1924).

I shall mention one more work from Vorovka’s short papers devoted to mathematics and its application. It is the paper titled **Poznámka o kauzálním myšlení***(A note on causal thinking)* (Ruch Filosofický, V, 1925). Its content is also part of the lecture read at the 6th Congress of Philosophers: **Analyse de la relation causale a l’aide du concept d’action suffisante** (published in Ruch Filosofický, VI, 1926-27). If we knew every instant state of the world into all smallest details and if we could predict the process of all changes, it would not make much sense to think whether some event is a cause of another one. We would see everything and there would be no need to explain. But the world is immensely complicated. We cannot observe and investigate natural events in another way than to choose only a limited section of only several events. And thus, as a result of the impossibility of the fully universal conception, we are necessarily led to the explanation that one thing influences another one. The concept of influencing, says Vorovka, cannot be eliminated from science.

I would not like to forget one Vorovka’s interesting paper on mathematical education at Czech secondary schools. This paper, written in German, was published in Vienna in 1914 as a part of a great report of the international committee (*Berichte über den mathematischen Unterricht in Österreich*). Apart from historical notes on old textbooks Vorovka presents a very thoroughly worked out survey of new directions which were introduced into mathematics teaching after the curriculum reform of 1909. He goes through the methods of individual textbooks and compares them to methods used in other countries. At the end of the work, he names K. Osovský as a collaborator. This booklet will be an excellent help for those who are interested in the history of mathematics teaching in our schools.

Vorovka’s active mind was not occupied by the problems of philosophy of mathematics all his life but later he turned to general philosophical mysteries. His activities in this field will be dealt with by a colleague from the Faculty of Arts. I shall finish my recollections with some more words. Vorovka should be praised for having brought Poincaré’s thinking closed to us. He did so in his work on an opinion in mathematics in a way that lively recalls all excellent qualities of his bright mind. In the foreword, he apologizes to the reader in the following way: ”Although I am greatly aware of the problematic nature of my results, all the same I presented them in a simple form as it is more likely to arise an interest in the matter.” Those who knew Vorovka personally, will not forget his inborn modesty which led him to criticism of his own opinions that may be sometimes harsch and reckless. They will understand that Vorovka was able to sincerely apologize for using an indicative in his philosophical deductions. In the dispute between psychologism and logicism, he opted for the first; however, he believed that philosophical problems are eternal and he tried to formulate precisely his reasons which led him to psychologism, as well as to analyze even the directions of thinking he disagreed with, in order to extract what is right in them. On January 9th, 1918 a meeting devoted to a discussion on Vorovka's work **Úvahy o názoru v matematice** *(Considerations on an opinion in mathematics)* was held. A number of lecturers read their opinions. Some welcomed psychologism, others could not accept what Vorovka wrote about logicism; some looked at the questions from mathematical point of view, others started a discussion on philosophical problems and all together indicated that publishing the **Úvahy** was a significant event. At the end of the meeting, Vorovka answered every questions. It was a really beautiful evening which the participants like to recall. Vorovka himself had the best remebrance of it too.

Vorovka wrote in the study I have been writing about before (Česká Mysl, XIV, p. 30) that in modern times "It is not philosophers who bring problems to mathematicians, but often mathematicians to philosophers. And even more: exact sciences reached a complete autonomy and they solve border problems with their own methods independent of philosophical systems". What difficult tasks are ahead of those who explore mathematical-philosophical problems in the sense of Vorovka! Some have already said that there is nobody can replace Vorovka. Unfortunately, this is more than true. Which one of those who have on mind mainly philosophical mysteries will study mathematical and physical sciences and will follow their advancements in the same way as Vorovka who after graduating from the university continued to study various problems of mathematics, atom theory, the most difficult parts of electricity theory and others? And which one of those who are interested mainly in mathematical problems is able to engage himself in philosophical studies and to gain such a scope in philosophy, its history as well as history of mathematics and physics as Vorovka had? Now, when the charm of his noble and fine personality cannot affect anybody directly any more after Karel Vorovka's early passing, his followers can learn about his thinking and work in the philosophy of mathematics from his literal work only. These are not many pages, but they are profound manifestations of subtle intelligence and proofs of careful and conscious work, an example for everybody who is going to be engaged in questions lying on the borders between mathematics and philosophy.

ANNEX 2

A note to the Problem of Gambler's Ruin

Karel VOROVKA

Translation from the Czech by Stepanka Bilova and Laurent Mazliak

At the end of his work *De ratiociniis in ludo aleae* Huygens states five exercises whose solutions were given by Jakub Bernoulli in his great and famous book *Ars conjectandi*. The last of those exercises is also a simple exercise known with the name "lasting of the game" or "Gambler's Ruin".

If two gamblers A and B play with the probability of winning a game p and q, and if we know their wagers a and b as well as their capitals x and y, then two questions are especially interesting: First we can calculate the probability that A will be ruined in the game that can be repeated in an unlimited number of times until one of the gamblers loses all his property. And the other question may be asked what is the probability that Gambler A will be ruined within a certain finite number of games.

We shall deal with the first case.

As both capitals of the gamblers, varying during the game, complement each other to make a constant sum s, we can, on the basis of the equation $x + y = s$, consider the probability of ruining any gambler to be a function of one variable x or y as we like.

Thus let the probability of Gambler A's ruin be given by an unknown function $u(x)$, and of Gambler B's ruin by $v(x)$.

J. Bernoulli calculated these functions in a special case of Huygens's exercise which supposes equal wagers, and then checked additionally by a calculation that the equation $u(x) + v(x) = 1$ is identically satisfied, evidently meaming the certainty that with the unlimited duration of the game one or the other gambler will be ruined.

As Todhunter[6] states this result could be expected as the common sense suggests.

This principle was also adopted *without proof* by De Moivre in his work *De Mensura Sortis* (Probl.IX) and in his book *The Doctrine of Chances*(probl.VII). Todhunter[7] points out that the solution given by De Moivre was not complete. That is, using an ingenious method, De Moivre found the proportion of the probabilities of both gamblers and then without proof he accepted for certain that an infinite series of games results in one of the gamblers' ruin. Then, however, he could immediately deduce also the exact values of both probabilities.

In his classical work Laplace gives a detailed analysis of the problem of the lasting of the game dealing mainly with its more difficult second part if the number of played games is finite. But the fact that one or the other gambler will certainly be ruined with the unlimited duration of the game is passed over in silence by him.

In his book on probability Poincaré[8] presents only Bernoulli's simple calculation. He, however, stresses that the possibility of continuing the game indefinitely with continual balancing of winnings and losses was not excluded a priori.

I found a real general proof that one or the other gambler will be certainly ruined only Bertrand's book[9]. This proof is conducted only with words without calculations however it does not lose anything from its exactness in this way.

It contains the following consideration: If the amount of games is great, then also the more probable combination has a very small probability which is inversely proportional to the square root of the total number of games according to Stirling's formula. The same will be especially more true about every other combination or also about every finite group of combinations unless, however, the number of combinations contained in the group increases infinitely together with the number of games. With a fixed total number of games, the number of games won by one gambler may, however, oscillate only in such boundaries that neither gambler loses more than their initial property allows. The difference between these two boundaries is constant and it is completely independent of the total number of games. Thus if the total number of games grows to infinity, the probability of a limited number of combinations decreases under any boundary. In

[6]Todhunter, A History of the Mathematical Theory of Probability, p.63

[7]ibid., p.147

[8]Poincaré, Calcul des Probabilités, p.48

[9]Bertrand, Calcul des Probabilités, p.105

other words: One of both gamblers will certainly be ruined within an unlimited duration of the game.
Only one thing can be objected to this proof - it is that it proves more than necessary; as it concerns such a way of playing that only the final balance would decide about the ruin, while in reality a gambler can be ruined by several initial losses.
I suppose that the nature of the proof is best reflected in the following proof based on the linear equation with finite differences

$$u(x) = pu(x+b) + qu(x-a)(1)$$

which is easy to deduce from compound probability principles. This equation is of order m where $m = a + b$ and we integrate it in a way analogical to linear differential equations with constant coefficients. We put $u(x) = \alpha^x$ and we obtain the general integral

$$u(x) = C_1\alpha_1^x + C_2\alpha_2^x + \cdots + C_m\alpha_m^x,$$

in which $\alpha_1, \alpha_2, \ldots, \alpha_m$ denote the roots of the equation

$$p\alpha^{a+b} - \alpha^a + q = 0.(2)$$

This equation has always the root $\alpha_1 = 1$ because $p + q = 1$. This root can, however, be a double root. It becomes so in the case when the game is fair, i.e. the wagers are in the rate of the probabilities

$$a : b = p : q.$$

There are no other multiple roots. Thus, the general integral can have either the form

$$u(x) = C_1 + C_2\alpha_2^x + C_3\alpha_3^x \cdots + C_m\alpha_m^x$$

if the game is unfair, or

$$u(x) = C_1 + C_2x + C_3\alpha_3^x \cdots + C_m\alpha_m^x$$

if the game is fair.
Bertrand keeps only the first two terms of the integrals, thus the two conditions

$$u(0) = 1 \text{ and } u(s) = 0$$

meaning the certainty, or the impossibility of the ruin respectively allow him to determine the constants.
Bertrand's method is not absolutely complete as A. Markov[10] pointed out. We must, however, observe that Bertrand mentions how his formulae change if the gambler has not lost all his fortune, but has not a sufficient amount for the wager.
If we work with the complete integral, we must have further $m - 2$ conditions to determine all m constants. They can be of two types according to the game practice. A gambler is considered ruined either in the sense of Bertrand's note if the rest of his property is smaller than the wager, or if he is in this case allowed to continue playing and gets into debt in case of another lost game.
In the first case we get the conditions

$$\begin{matrix} u(0) = 1, u(1) = 1, \ldots u(a-1) = 1, \\ u(s) = 0, u(s-1) = 0, \ldots u(s-b+1) = 0; \end{matrix} \quad (3)$$

[10] Bulletin de la Société phys.-math. de Kas[!]an, 1903.

in the second one the conditions are

$$u(0) = 1, u(-1) = 1, \dots u(-a+1) = 1,$$
$$u(s) = 0, u(s+1) = 0, \dots u(s+b-1) = 0.$$

It is not important which conditions we use for the following considerations. Let us choose the first set of conditions as Markov did. It would be, however, complicated to determine the roots of the equation (2), and it is therefore preferable to use another way for finding an approximate value instead of the exact expression for the function $u(x)$. This is what the paper published by Markov is about.
We can use the exact expression to prove the certainty of the ruin though.
We calculate the probability of Gambler B's ruin in the same way as for Gambler A. To determine the function $v(x)$ we get

$$v(x) = pv(x+b) + qv(x-a), (4)$$

which is completely identical to the equation for $u(x)$. Its integral will have the same form too, either

$$v(x) = D_1 + D_2\alpha_2^x + D_3\alpha_3^x \cdots + D_m\alpha_m^x$$

or

$$v(x) = D_1 + D_2 x + D_3\alpha_3^x \cdots + D_m\alpha_m^x$$

Only the constants $D_1, D_2, \dots, D_m$ will be determined by different conditions, i. e.

$$\begin{aligned} &v(0) = 0, v(1) = 0, \dots v(a-1) = 0, \\ &v(s) = 1, v(s-1) = 1, \dots v(s-b+1) = 1; \end{aligned} \quad (5)$$

As both equations for $u(x)$ and $v(x)$ have identical shapes, we can obtain a new equation by summing

$$f(x) = u(x) + v(x)$$

giving the probability that one or the other gambler will be ruined. We have the following equation for the function

$$f(x) = pf(x+b) + qf(x-a), (6)$$

whose general integral will be either

$$f(x) = E_1 + E_2\alpha_2^x + E_3\alpha_3^x \cdots + E_m\alpha_m^x$$

or

$$f(x) = E_1 + E_2 x + E_3\alpha_3^x \cdots + E_m\alpha_m^x$$

The conditions for determining the constants follow from summing the conditions (3) and (5) for $u(x)$ and $v(x)$. Thus we get the following conditions

$$\begin{aligned} &f(0) = 1, f(1) = 1, \dots f(a-1) = 1, \\ &f(s) = 1, f(s-1) = 1, \dots f(s-b+1) = 1; \end{aligned} \quad (7)$$

Let us suppose for instance. a fair game and proceed to the solution of the corresponding system of linear equations

$$\begin{array}{lllll} 1 & = & E_1 & + & 0 + E_3 + \dots E_m \\ 1 & = & E_1 & + & E_2 + E_3\alpha_3 + \dots + E_m\alpha_m \\ \dots & \dots & \dots & \dots & \dots \\ 1 & = & E_1 & + & E_2(a-1) + E_3\alpha_3^{a-1} + \dots + E_m\alpha_m^{a-1} \\ 1 & = & E_1 & + & E_2 s + E_3\alpha_3^{s} + \dots + E_m\alpha_m^{s} \\ 1 & = & E_1 & + & E_2(s-1) + E_3\alpha_3^{s-1} + \dots + E_m\alpha_m^{s-1} \\ \dots & \dots & \dots & \dots & \dots \\ 1 & = & E_1 & + & E_2(s-b+1) + E_3\alpha_3^{s-b+1} + \dots + E_m\alpha_m^{s-b+1}. \end{array}$$

As the determinant of this system contains only units in the first column, we obviously get $E_1 = 1$, and

$$E_2 = E_3 = \dots = E_m = 0$$

thus identically

$$f(x) = 1.$$

It follows that the equation

$$u(x) + v(x) = 1$$

is satisfied identically and that one of the gamblers will *certainly be ruined* is the duration of the game may be unlimited. This proof is based on the fact that 1 is always one root of the equation for α. The proof would be conducted in a similar way if the game were unfair or if the gamblers were considered ruined only after running into debt.