Journ@l Electronique d'Histoire des Probabilités et de la Statistique

Electronic Journ@l for History of Probability and Statistics

Vol 8; Décembre/December 2012

www.jehps.net

Paolo Ruffini (1765-1822)

Franca CATTELANI DEGANI[1]

Paolo Ruffini was among the brightest lights of the town of Modena in Italy even if he was only an adopted citizen; in reality, he spent his whole life there, except for his first years which were spent between Valentano (Viterbo), where he was born in 1765, and Reggio Emilia, the original town of his family.
He studied at the University of Modena, at that time the main town of the Este Dukedom, and, following his father's steps, he took a degree in medicine in 1788. From 1791, he began this profession, providing a lot of help to his fellow-citizens, although he was the duchess' personal physician.

However, Paolo Ruffini is now mostly known as a mathematician. He received his education in mathematics during the so called « two years philosophical period», a period of studies preceding the choice of the specialized faculty, and was introduced to sublime calculus and the algebraic equations' theory by Paolo Cassiani (1743-1806), a Modenese lawyer who had been the first teacher of mathematical analysis at the local university, when it reopened in 1772. In 1787, Cassiani was forced to renounce teaching on account of political appointments and he was replaced by Ruffini, who was still a student. So began for him the academic activity of professor of mathematics, which he continued until the eve of his death with only one interruption in 1798-99, when he was barred from teaching because he had refused to take the oath of fidelity to the Republic, established during the French occupation. In his lasting career, he held chairs of every mathematical branch at the time and at the different local institutions: the University (1787-1798, 1799-1803), the Departmental Lyceum, into which the University had been transformed in Napoleonic time (1803-1807), the military School of

1 franca.cattelani@unimore.it. F.Cattelani Degani was professor at the University of Modena and Reggio Emilia (Italy).

Engineers and Artillery, established in Modena following Napoleon's desire (1807-1814), again the University, after the fall of the French Empire (1814-1822).

With the Restoration in 1814 began Ruffini's period of highest celebrity: in the restored University, the Duke not only granted him several chairs (Applied Mathematics at the Philosophical Faculty and Practical Medicine at the Medical Faculty, added by Clinical Medicine in the following year), but also the appointment of rector.

His great renown in the sciences resulted in his election in 1816 as President of the Italian Society of Sciences (later said `of the XL'); from 1803 he was a member of the National Institute and from 1806 of the Catholic Religion's Academy (the present Thomas Aquinas' Academy).

He died in Modena in 1822.

Paolo Ruffini achieved important results in the algebraic field, results mostly related to his first publication: *General theory of equations in which it is shown that the algebraic solution of the general equation of degree greater than four is impossible* (*Teoria generale delle Equazioni, in cui si dimostra impossibile la soluzione algebraica delle equazioni generali di grado superiore al quarto*) (1799). In this voluminous treatise, he explained all that was known at the time about the algebraic equations and, as he himself recognized, he started from results of J.L. Lagrange (1736-1813), but he was able to go beyond.

For the cubic equations he solved the old problem about the irreducible case, showing that it is not possible to express the three real roots without the use of complex numbers. For the equations of higher degrees, contrary to what is said in the title of the treatise, he really provided the first proof that quintic equations cannot be solved by radicals, but made only mention to a possible extension to higher degrees.

The result was strikingly opposed to what the scientific community expected, generally convinced that it was possible to find solving formulas for equations of degree greater than four, and so it was received with no little skepticism, probably also because Ruffini or his advisor Cassiani were hardly known out of the Este Dukedom.

Ruffini reelaborated several times his proof (1803, 1806, 1813) either for completing or simplifying it. The difficult reading was caused from the fact that during his long and laborious elaboration he had been forced to build, step by step, the basic elements of group theory, which today serve as a tool for the proof. During several years, he hoped in vain that his proof would be read and judged by Lagrange, by the Academy of Sciences in Paris or by German

mathematicians. Only in September 1821, a few months before he died, had he the honour of receiving a letter from A. Cauchy (1789-1857), where the latter acknowledged the importance and the correctness of the proof. When Cauchy sent his letter, he had already written an important work on permutation groups (1813-1815) for which he had been influenced by Ruffini's results, some of which he generalized.

For a long time, the theorem of algebraic insolubility for the equations of degree greater than four was known simply as *N. Abel's theorem* as the Norwegian mathematician had proved it in 1825, using the theorems that Cauchy had obtained as a consequence of the results of the Modenese mathematician.

Later the works of H. Burkhardt (1892) and above all of E. Bortolotti (in the first half of 20th century) attracted a new interest for Ruffini's achievements.

Ruffini's name is also connected with a method, still employed, to identify and to approximate numerically the roots of equations: it is the so-called *Ruffini – Horner's method*, though the works of W.G. Horner (1786-1837) are dated 1819 and 1845, while Ruffini's memoir was published in 1804. After Horner's publications, this method had a large diffusion, in England especially, thanks to the many articles of A. De Morgan and J. R. Young, while Ruffini's name was forgotten. At last, in 1911, F. Cajori pointed out that Ruffini had anticipated Horner's method by fifteen years and, from then on, it was mentioned under both names. The idea of the method is to compare the signs of the polynomial $f(x)$ for successive integers values of x. When $f(a)$ and $f(a+1)$ have opposite signs, there is a root r between a and $a+1$. Ruffini introduced an algorithm based on successive divisions of the polynomial $f(x)$ by $x-a$ (the *Ruffini's rule*) in order to obtain the coefficients of the polynomial $g(x) = f(x+a)$ which now has the root $r-a$ comprised between 0 and 1; an obvious transformation allows us to obtain the coefficients of the polynomial $h(x) = g(x/10)$ for which $10(r-a)$ is a root comprised between 0 and 10. Then the same method is applied to $h(x)$ and this recursive application allows us to obtain the decimals of the root r.

Ruffini was also a fervent Catholic and already with his refusal to take the oath to the Republic (1798) he had given a proof of his inflexible respect for his principles, because he had not been allowed to add to the official formula that «it was intended to respect and save Religion» («intendevasi rispettata e salvata la Religione»). With an evident apologetic purpose, he wrote three works on philosophical subjects.

In the first work, *The immateriality of the soul* (*Immaterialità dell'anima*) (1806), he followed an argument using the typical methodology of a proof of a mathematical theorem in order to prove «that a Being endowed with the

knowing faculty is necessarily immaterial» («che un Essere dotato della facoltà di conoscere è necessariamente immateriale»); he opposed some principles exposed by Erasmus Darwin (1731-1802) in his «*Zoonomia*» (1794–1796), a work in which the latter had anticipated the theories on evolutionism, which afterwards were resumed by his grandson Charles Darwin, and in which he had stated the materiality of every being endowed with will and intelligence.

In the third work, *Concerning the definition of Life given from Brown* (*Intorno alla definizione della vita assegnata da Brown*), which was published posthumously (1833), but already read in 1818 at the Catholic Religion's Academy, he criticized conceptions leading to materialism.

The second philosophical work is *Critical considerations on the philosophical essay about probability of Count Laplace* (*Riflessioni critiche sopra il saggio filosofico intorno alla probabilità del signor Conte Laplace*) (1821) and, as we can infer from Ruffini's letters, he began to work on it under pressure from some fellow-citizens, especially the Marquis Gherardo Rangone (1744–1815), a learned man, with various interests, who had founded a scientific academy in his home (1783-1791) and who shared with Ruffini the zealous defense of Catholic faith.

The *Critical considerations* contain four essays, each divided in two parts, where the author began translating some parts of Laplace's *Essai philosophique* that he had intended to oppose.

In the first essay, *Remarks about the principles that Count Laplace establishes for the applications of the calculus of probability* (*Osservazioni intorno ai principj che stabilisce il sig. conte Laplace per le applicazioni del calcolo delle probabilità*), Ruffini starts rejecting the conception of the so-called «Laplace's intelligence» («*intelligenza di Laplace*»), because the acceptance of a strict determinism for a natural phenomenon with a strong concatenation between causes and effects does not leave any space to the free will.

In the second essay, *Considerations about the laws of probability which result from the infinite multiplication of the events* (*Considerazioni intorno alle leggi di probabilità che risultano dalla moltiplicazione infinita degli eventi*), Ruffini refused to use the scheme of drawing from urns to deal with the probability of a natural phenomenon as an application of the law of large numbers to infer its regularity, a fact which would contradict its dependence on a Universe regulating Providence. Moreover, as a believer as well as a mathematician, he also thought unacceptable to subject moral issues to calculus, anticipating many criticisms raised against Laplace later in the nineteenth century.

In the third essay, *Considerations about Count Laplace's hypotheses for the origin of the planets and the comets* (*Considerazioni intorno alle ipotesi del*

signor Conte Laplace per l'origine dei pianeti e delle comete), he maintained that the «primitive Cause», which according to Laplace is the basis of the planetary motions, is nothing but a «supreme Intelligence» or a «supreme Creator». He thought that Laplace or F.W. Herschel (1738-1822), in their analogous conceptions of the universe and of the heavens, cannot possibly get rid of admitting a «Moderator of everything », that is to say God.

The fourth and last essay is entitled *About the principles considered by Count Laplace with regard to the probability of evidence* (*Intorno ai principj ritenuti dal Sig. Conte Laplace relativamente alla probabilità delle testimonianze*) and studies the case of one or more witnesses. Laplace's aim was to prove the unreliability of evidence for unusual events or miracles. Ruffini contested that one could measure the degree of reliability of a witness as if it were a question of draws from an urn: to deceive, voluntarily or involuntarily, depends on a man's will and the circumstances of his life, neither susceptible to measurement. Besides, even if Ruffini conceded that one could really compute the probability that a witness be honest or dishonest, he thought that Laplace had made a mistake because he looked at events which had already happened and were well known to the witness as if they were about to happen and unknown to the witness.

As already said, Ruffini wrote his *Critical considerations* also on account of pressures made on him by several persons, who - on one hand - were afraid of the attack presented in the *Essai philosophique* against the existence of free-will and – and on the other hand – found in Ruffini two qualities to be able to oppose a scientist of such great renown as Laplace:

1. his ability to lead a controversy for the defense of the principles of the Catholic faith (for his work about *The immateriality of the Soul* Pope Pio VII had already granted him with a gold and a silver medals);
2. his recognized mathematical and scientific competence, highlighted by his presidency of the Italian Society of Sciences.

However, Ruffini would not allow his propensity for controversy full reign and it is very probable that Laplace never knew about Ruffini's *Critical considerations*, even if a short review was published in Paris (1822) in the journal *L'ami de la religion et du roi; journal ecclésiastique, politique et littéraire (*21, fasc. 791, 122-123).

Works of Paolo Ruffini

1 - *Teoria generale delle Equazioni in cui si dimostra impossibile la soluzione algebraica delle equazioni generali di grado superiore al quarto,* Bologna,

Stamp. S.Tommaso d'Aquino, Voll. 2;
2 - *Della soluzione delle equazioni algebraiche determinate particolari di grado superiore al quarto,* Mem. Soc. It. delle Scienze, T. IX (1802), 444-526;

3 - *Riflessioni intorno alla rettificazione ed alla quadratura del circolo,* Mem. Soc. It. delle Scienze, T. IX (1802), 527-557;

4 - *Della insolubilità delle equazioni algebraiche generali di grado superiore al quarto,* Mem. Soc. It. delle Scienze, T. X, P. II (1803), 410-470;

5 - *Sopra la determinazione delle radici delle equazioni numeriche di qualunque grado,* Modena, Soc Tipografica, 1804;

6 - *Risposta di P. Ruffini ai dubbi propostigli dal Socio Gianfrancesco Malfatti sopra la insolubilità delle Equazioni di grado superiore al quarto,* Mem. Soc. It. delle Scienze, T. XII, P. I (1805), 213-267;

7 - *Riflessioni di P. Ruffini intorno al metodo proposto dal Consocio Gianfrancesco Malfatti per la soluzione delle Equazioni di quinto grado,* Mem. Soc. It. delle Scienze, T. XII, P. I (1805), 321-336;

8 - *Della immaterialità dell'anima,* Modena, Eredi Bart. Soliani, 1806;

9 - *Della insolubilità delle equazioni algebraiche generali di grado superiore al quarto qualunque metodo si adoperi algebraico esso siasi o trascendente,* Mem. dell'Istit. Naz. Italiano, Classe di Fisica e Matematica, T. I, P. II (1806), 433-450;

10 - *Alcune proprietà generali delle funzioni,* Mem. Soc. It. delle Scienze, T. XIII, P. I (1807), 292-335;

11 - *Algebra e sua Appendice,* Modena, Soc. Tipografica, Voll. 2, 1807-08;

12 - *Di un nuovo metodo generale di estrarre le radici numeriche,* Mem. Soc. It. delle Scienze, T. XVI, P. I (1813), 373-429;

13 – *Riflessioni intorno alla soluzione delle equazioni algebriche generali,* Modena, Soc. Tipografica, 1813;

14 – *Memoria del tifo contagioso,* Mem. Soc. It. delle Scienze, T. XVIII, P. I, fasc. II (1820), 350-381;

15 – *Intorno al metodo generale proposto dal Signor Hoêne Wronski onde risolvere le equazioni di tutti i gradi,* Mem. Soc. It. delle Scienze, T. XVIII, Fasc. I di Matem.(1820), 56-68;

16 – *Della classificazione delle curve algebriche a semplice curvatura,* Mem.

Soc. It. delle Scienze, T. XVIII, P. I di Matem.(1820), Opus. I: 69-142; Opus. II: 269-396;

17 – *Riflessioni critiche sopra il saggio filosofico intorno alla probabilità del signor Conte Laplace,* Modena, Soc. Tipografica, 1821;

18 – *Elogio di Berengario da Carpi,* Modena, Fasti Letterari della Città di Modena e Reggio, T.III, (1824) (postuma);

19 – *Alcune proprietà delle radici dell'unità,* Mem. dell'I.R. Istit. del Regno Lombardo Veneto, T.III, (1824) (postuma);

20 – *Osservazioni intorno al moto dei razzi alla Congreve,* Mem. R. Accad. di Sc. Lett. e Arti di Modena, T.I, (1833), 56-78 (postuma);

21 – *Riflessioni intorno alla eccitabilità, all'eccitamento, agli stimoli, ai controstimoli, alle potenze irritative, alle diatesi sì ipersteniche che ipostenichе,* Mem. R. Accad. di Sc. Lett. e Arti di Modena, T.I, (1833), 1-55 (postuma);

22 – *Intorno alla definizione della vita assegnata da Brown,* Mem. R. Accad. di Sc. Lett. e Arti di Modena, T.I, (1833), 319-333 (postuma);

23 – *Macchina atta a contenere le fratture oblique del femore in modo da impedire l'accorciamento della coscia,* mss. in Filza 17, Archivio Ruffini, Accademia Nazionale Sc. Lett. Arti di Modena;

24 – *Memoria sul principio delle aree,* mss. in Filza 17, Archivio Ruffini, Accademia Nazionale Sc. Lett. Arti di Modena.

Bibliografy

G. Barbensi, *Paolo Ruffini*, Modena, Accademia di Scienze Lettere ed Arti, 1956;

F. Barbieri, *Il pensiero di Paolo Ruffini su alcuni problemi epistemologici,* Giorn. Fis., 32 (1991), 167-174;

F. Barbieri – F. Cattelani Degani, *Catalogo della corrispondenza di Paolo Ruffini,* Modena, Accademia Nazionale di Scienze Lettere ed Arti, 1997;

F. Barbieri – C. Fiori, *Paolo Ruffini all'Università di Modena,* Atti del Convegno di studi in memoria di Giuseppe Gemignani, Modena, Accademia Nazionale di Scienze Lettere e Arti, 1995, 93-119;

E. Bortolotti (a cura di), *Opere matematiche di Paolo Ruffini,* T. I, Palermo,

Tip. Matematica, 1915; T. II e III, Roma, Ed. Cremonese, 1953 e 1954;

H. Burkhardt, *Die Anfange der Gruppentheorie und Paolo Ruffini,* Zeitschr. fur Math. und Physik, Jahrgang XXXVII (1892); Italian translation by E. Pascal, Ann. Mat. Pura Appl., (2) 22 (1894), 175-212;

F. Cajori, *Horner's method of approximation anticipated by Ruffini*, Bull. Amer. Math. Soc., 17, Num. 8 (1911), 409-414;

E. Carruccio, *Paolo Ruffini matematico e pensatore,* Atti Mem. Acc. Sc. Lett. Arti Modena, (6) 8 (1966), Atti Acc., LIII-LXIX;

J. Cassinet, *Paolo Ruffini (1765-1822): la résolution algébrique des équations et les groupes de permutations,* Boll. Stor. Sci. Mat., 8(1988), n.1, 21-69;

R. Franci, *L'algebra in Italia dal 1799 al 1813,* Physis, 29 (1992), n.3, 745-770;

A. Lombardi, *Notizie sulla vita e su gli scritti di Paolo Ruffini*, Modena, Tip. Camerale, 1824;

G. Varoli, *Su un'opera pressoché sconosciuta di Paolo Ruffini «Riflessioni critiche sopra il saggio filosofico intorno alle probabilità del Sig. Conte Laplace»,* Statistica, 17 (1956), 421-442.